

## **Research on the Trading Strategy Based On Interest Rate Term Structure Change and Pricing Error**

Lu Shen

*School Of Economics and Business Administration, Central China Normal University, 430079 Wuhan, Hubei Province, China*

---

**Abstract:** *Bond pricing errors exist in the bond market universally, the formation of the reasons for its formation has been controversial. In this paper, in order to obtain the pricing error, the authors first estimate the term structure of interest rate of China's interbank market by using the three spline model and the Svensson model. Then, the author using the moving average model and time series model to build the bond trading strategy based on the pricing error. Through the simulation of our bond portfolio trading, the result shows that bond trading can obtain about 11 basis points of the annual excess return based on bond pricing errors, and the excess return rate is not caused by different bond liquidity or risk characteristics, instead is due to the effective economic information included in the bond pricing error.*

**Keywords** -Pricing error; term of interest rate; trading strategy; economic information

---

### **I. Introduction**

The importance of the interest rate of financial market is self-evident, in an efficient market, the accurate estimation of interest rates term structure can reasonably explain the information contained in the bond market prices, for the same credit rating for any period of time. However, almost all empirical studies on the pricing of bonds at home and abroad show that there is a significant difference between the theoretical price and the market price of the bond (Svensson, 1994; Jankowisch and Pichler, 2004). So what caused this wide spread? Some scholars will cause attributed the spread of bonds of different tax rate and different mobility, but Elton (1998) and Jankowisch (2006) pointed out that these two factors cannot fully explain the observed spreads. Therefore, the spread is bound to a certain extent by the model setting error or by the market information on the pricing errors caused by.

The purpose of this paper is to analyze whether the observed pricing error contains economic information. If the pricing error in a certain degree of market information is caused by the price of the market information, then based on these are not reflected in the model of market information to be able to get the excess return on the transaction. Flavell et al. (1994), sercu (1997) and ioannides (2003) through the empirical research by using Belgium and the UK bond market data, utilize the pricing error trading strategies can get excess returns of 50 basis points rate. But because it does not contain any transaction costs and of different interest rate risk income just by the same method adjusted; therefore, the excess rate of return can only reflect the bond pricing error may contain effective market information.

According to the deficiencies of the previous literature, this paper from the reality of the pricing error to develop bond trading strategies, fully consider the transaction cost, for the first time in China is proposed based on bond pricing error of the idea of bond trading, and the use of practical methods of risk adjusted to correct for portfolio under different interest rate risk return. In this paper, the transaction decision-making is determined by two factors, one is the average rate of interest rate theory, and the other is to issue bonds to sell or buy signal after calibration of the pricing error time series model. In the construction of this trading strategy based on. Firstly, this paper uses transaction data of China's Shanghai Stock Exchange Market from January 2010 to December 2014. Also, McCulloch (1975) proposed by cubic spline model to estimate the term structure of interest rates, and then get the pricing error of each country's debt. In order to ensure the trading strategies rely on bond pricing error, we make estimation of the term structure of the same rate according to extended model based on Svensson (1994) and Nelson-Seiglon (1987). In the two models of term structure of interest rates of national debt pricing, and according to the bond pricing residuals obtained by the two models a larger proportion is on the basis of the same interval, and then based on pricing residuals of bond trading.

The remainder of this paper is organized as follows. In the second part, estimation of term structure of interest rates, and access to the Chinese treasury bond pricing error; the third part based on bond pricing error of simulated trading; the fourth part based on transaction pricing error of the result to carry on the analysis; the fifth part is the conclusion.

## II. Bond Pricing And Pricing Error

For any of the bond in market, estimation of term structure of interest rate  $r(t)$  or the discount factor  $D(t)$  should be able to accurately reflect the market price of the bond, then in order to observable to bond market price  $P$  and the duration of the bond cash flow  $Z$  conditions, there is the following relationship:

$$P = \sum_j Z_j \cdot D(T_j) + \varepsilon \quad (1)$$

Among them,  $\square$  represents a bond pricing error; Further, pricing factor  $D(t)$  usually are regarded as bonds with a maturity of continuous function, depends only on the number of parameters, the pricing error epsilon squared and minimized to calibrate the parameter estimates. If the  $\square$  is a free parameter in the pricing factor function, there are:

$$D(t) = f(t; a) \quad (2)$$

And

$$P = \sum_j Z_j \cdot f(T_j; a) + \varepsilon \quad (3)$$

In order to obtain accurate bond price theory, we must first equal credit level of the term structure of interest rates has accurate estimation. Therefore, this paper selects is widely used or graphic description ability of McCulloch (1975) three times spline model and Svensson (1994) model were estimated term structure of interest rate, and obtains the theoretical price of the bonds.

McCulloch (1975) used the three spline method is to  $n$  only bonds from small to large order, the use of  $k-1$  nodes will be the longest period is divided into  $K-2$  range, so as to get  $k$  parameters. Noded  $j = m_h + \theta(m_{h+1} - m_h)$ , ( $j=1, 2, \dots, k-1$ ), among them, the integer part  $h = (j-1)n/(k-2)$ ,  $\theta = (j-1)n/(k-2) - h$ , and  $d_0 = d_1 = 0$ ,  $d_{(k-1)} = d_k = m_n$ . Then the basis function set by the McCulloch is:

$$\begin{cases} 0 & m < d_{j-1} \\ \frac{(m-d_{j-1})^3}{6(d_j-d_{j-1})} & d_{j-1} \leq m < d_j \\ \frac{(d_j-d_{j-1})^2}{6} + \frac{(d_j-d_{j-1})(m-d_j)}{2} + \frac{(m-d_j)^2}{2} - \frac{(m-d_j)^3}{6(d_{j+1}-d_j)} & d_j \leq m < d_{j+1} \\ (d_{j+1} - d_{j-1}) \left( \frac{2d_{j+1}-d_j-d_{j-1}}{6} + \frac{m-d_{j+1}}{2} \right) & m \geq d_{j+1} \end{cases} \quad (4)$$

When  $j = k$ ,  $f_k(m) = m$

The calculated  $f_j(m)$  into the formula (3), resulting in the bond I pricing model

$$P_i = \sum_j Z_{ij} \cdot \sum_k a_k \cdot f_k(T_{ij}) + \varepsilon_i = \sum_k a_k \cdot \sum_j Z_{ij} \cdot f_k(T_{ij}) + \varepsilon_i \quad (5)$$

Because formula (5) is a linear structure type of  $a_k$ , then we can get the value of  $a_k$  by least square estimation. Because the model can also lead to the pricing error, in order to ensure that the pricing error is not entirely caused by the model, this paper uses Svensson (1994) model to estimate the term structure of interest rate again:

$$r(t) = \beta_0 + \beta_1 \left( \frac{1 - \exp\left(-\frac{t}{\tau_1}\right)}{\frac{t}{\tau_1}} \right) + \beta_2 \left( \frac{1 - \exp\left(-\frac{t}{\tau_1}\right)}{\frac{t}{\tau_1}} - \exp\left(-\frac{t}{\tau_1}\right) \right) + \beta_3 \left( \frac{1 - \exp\left(-\frac{t}{\tau_2}\right)}{\frac{t}{\tau_2}} - \exp\left(-\frac{t}{\tau_2}\right) \right) \quad (6)$$

$\beta_1, \beta_2, \beta_3, \beta_4, \tau_1, \tau_2$  is to be estimated parameters, so that the pricing factor function is

$$f(T_{ij}; a) = [1 + r(T_{ij})]^{-T_{ij}} \quad (7)$$

In order to estimate the parameters in the two models, taking into account the national debt of our country whether liquidity or transaction size and other types of bonds than there are great advantages, and inter-bank bond market trading volume accounted for the proportion of all of our national debt transaction size close to 95%, so this paper chooses China inter-bank market bond transaction data of this theory to carry out an empirical test. Only the flow of treasury bonds to, but there is no but because of China's inter-bank bond market maker too easy to inquiry mechanism to promote the bond trading, so the bond market price fluctuations are not frequent, often appear for a few days prices unchanged, in order to ensure the pricing error effectively, the sampling period for a week, from wind database for China's inter-bank bond from January 1, 2010 to December 31, 2014; JCP 260 week of trading information, excluding the price during this period did not change more than 3 times, and the remaining time limit in 20 years or less (long-term debt limit is too low), and ultimately won 57 National 216 observation values. In this paper, the parameter estimation and Simulation of subsequent transactions are completed by Matlab software. Table 1 is the bond distribution in the observation period of China's interbank market.

Under the conditions of K=4, using McCulloch three times spline model to estimate China inter-bank bond market interest rate term structure, at the same time, using formula 6 as the Svensson model estimated term structure of interest rate of treasury bonds, and further through the bond market price (price+accrued interest) by subtracting the model to calculate the theoretical price, thus the pricing error of the two models can be obtained. Through the use of McCulloch cubic spline model and Svensson model by China inter-bank bond market every week Treasury data information repeated iterative estimation, and get the bonds in the sample every bond in each week of bond pricing error. Table 2 is the distribution of China's national debt pricing errors.

**Table 1.**The maturity distribution of the inter-bank market bonds in China

Weekly bond	deadlines	0-3 years	3-5 years	>5 years
average	117	15	27	78
minimum	67	0	9	59
Maximum	161	48	45	86

**Table 2.**The distribution of Chinese Treasury bond pricing error

Three spline	Mean	StdDev	Maximum	Minimum
Mean absolute pricing error	20.76	2.51	27.06	16.33
Maximum pricing error	100.61	12.68	130.45	77.10
Minimum pricing error	-48.38	16.67	-22.25	-100.97
Svensson model	Mean	StdDev	Maximum	Minimum
Mean absolute pricing error	18.77	2.66	29.08	14.38
Maximum pricing error	116.51	15.06	137.82	80.43
Minimum pricing error	-50.74	20.68	-14.98	-96.79

If the pricing error containing market information, then for the same bond, the use of cubic spline model and Svensson model will overestimate (underestimate) bonds, that is, due to the existence of market information is not reflected in the model, cubic spline model overestimated (underestimated) a bond, Svensson model should also will be overestimated (underestimated) with a bond. In order to verify whether there is enough market information in the pricing error, this paper calculates the hit rate of the bond pricing error estimated by the two models according to the method proposed by Bliss (1997). First, the pricing errors are classified into three intervals, interval (pricing error  $\square$  10bp), zero interval ( $-10 \square$  pricing error  $<$  10), negative interval (pricing error  $<$  -10). In the cubic spline model is regarded as the premise of the benchmark model, by calculating the pricing error of the two models give fall into the same interval probability, finally found two models are presented for the pricing error and incidence rate of 69%, although the results suggest that the model selection will indeed affect the bond pricing, but also reflect national debt pricing errors contain information of the market and effective.

In theory, the pricing error of bonds should be independent of each other in time series, but the pricing error of all bonds in this paper is highly correlated, and the first order autocorrelation coefficient of the bond is up to 0.8918. It can be found that the time series of pricing error is not in the vicinity of 0 fluctuations in the bond value is overvalued or undervalued will continue for more than half a year.

Is not difficult to find that the time series of bond pricing error has two effects; first, the level effect, different bonds with different maturities and liquidity, so high mobility bonds usually have a positive pricing errors, and illiquid bonds exist negative pricing errors, thus leading to the different bond pricing error that there is a difference in the level. But floating premium does not explain all of pricing error of volatility, pricing errors also reflects the second effect is pricing error around non zero mean fluctuations, the second effect may is the parameter estimation process caused by noise components, may also contains information related to the bond price deviation relative to the market in the short run. If bond pricing error in a certain extent containing to deviate from the market conditions in short-term economic and effective information, then in addition to its liquidity and maturity to the level of bonds on behalf of pricing error effect, bond pricing error above (below) the mean value can be regarded as overestimated (underestimated) bonds; thus appropriate trading this relatively overvalued or undervalued bonds will be able to fire the excess rate of return.

### III. Trading Strategy

The sample concentration bond pricing error exists not only height autocorrelation, and surrounds the non-zero mean fluctuations, but such as sercu& Wu (1996), ioannides (2003) directly based pricing residuals for constructing trading strategy is not appropriate, because even if the high liquidity of bonds are overvalued, keep persistent overbidding, is difficult in the short term to obtain excess return rate. Therefore, this paper based on the theory of the value of the bond pricing error, and re build trading strategy. In view of the autocorrelation of bond pricing errors, this paper first sets the trading signal by the k order of the pricing error.

$$\mu_{i,t} = \frac{1}{k} \sum_{j=t-k}^{t-1} \varepsilon_{i,j} \quad (7)$$

$$\sigma_{i,t} = \sqrt{\frac{1}{k-1} \sum_{j=t-k}^{t-1} (\varepsilon_{i,t} - \mu_{i,j})^2} \quad (8)$$

Where  $\varepsilon_{i,j}$  means the pricing error at  $j$  week of  $i$  bond, and then build the following trading signal multiplier  $m$  in a given situation:

When  $\varepsilon_{i,j} > \mu_{i,j} + m \cdot \sigma_{i,j}$ , bond I is overvalued;  
 When  $\varepsilon_{i,j} < \mu_{i,j} - m \cdot \sigma_{i,j}$ , bond I is undervalued.

This trading strategy applies only to information on the market mispricing, therefore the replacement signal transaction design method, the structure is more general trading strategies, namely the use of time series models predict pricing error, so as to obtain the trading signals. Because the price is highly correlated with the error, this paper uses the AR (1) model to forecast the bond pricing errors.

$$d_{i,t} = c + b \cdot d_{i,t-1} + u_{i,t} \quad (9)$$

$d_{i,t} = \varepsilon_{i,t} - \varepsilon_{i,t-1}$ , For any one week in the observation period  $t$ , this paper uses AR (1) to estimate the coefficient, then the first phase of the next week to predict the next  $T-1$ , then the outward phase prediction results for the:

$$\hat{d}_{i,t+1} = c_{i,t} + b_{i,t} \cdot d_{i,t} \quad (10)$$

The standard error is:

$$\hat{\sigma}_{i,t+1} = S_e \cdot \sqrt{1 + \frac{1+d_{i,t}^2}{t \cdot \sum_{k=1}^t d_{i,k}^2 - (\sum_{k=1}^t d_{i,k})^2}} \quad (11)$$

Where  $S_e$  is formula (11) regression standard errors, using this result, can obtain  $\hat{\varepsilon}_{i,t+1}$  and  $\hat{\sigma}_{i,t+1}$ , then:

When  $\varepsilon_{i,j} > \mu_{i,j} + m \cdot \sigma_{i,j}$ , bond I is overvalued;  
 When  $\varepsilon_{i,j} < \mu_{i,j} - m \cdot \sigma_{i,j}$ , bond I is undervalued.

The trading strategy is the same as the moving average method, but the confidence interval for the next phase of the forecast results by sub  $m$  is 1.96 and 1.65 represent 95% and 90% confidence intervals, respectively.

Under the premise of building a good trading strategy, trading rules is defined as follows: (1) in the 2010 any week trading, the first standardized bond portfolio was 100%; (2) when found a sell signal, sold all of the bond portfolio; (3) the selling income funds and emergency interest part of a buy signal was found in the bond investment, the remaining part of the investment in the inter-bank overnight lending market is surplus funds invest in Shibor overnight interest rates. Moreover, it does not allow short selling and margin trading, taking into account China's bond trading commission free, transaction cost is only every transaction amount of 0.0001%, equal to 0.01 percentage point, and the highest do not exceed 100 RMB, compared with the pricing error is still small, so all the bonds easily can ignore the cost. So in this transaction strategy under the combination of  $P$  weekly earnings can be expressed as formula:

$$r_i^P = \frac{\sum_{i=1}^{n,t-1} w_{i,t-1} \cdot P_{i,t} + NIC \cdot \left(1 + \frac{1}{360}\right) \cdot r_{t-1}^{MMY} + \text{Coupon}}{\sum_{i=1}^{n,t-1} w_{i,t-1} \cdot P_{i,t-1} + NIC} - 1$$

In order to make the simulation trading more close to the actual transaction, this article provides that the transaction duration is greater than one day, and then avoid the excess return of the accidental factors. In addition, this paper will bond between the inter-bank bond index and the maturity of the portfolio as a benchmark, which holds a combination of the current market in the current market by all the circulation of treasury bonds, according to the weight of the weight of investment. Will be based on bond pricing error exchange obtained from the expected return on the benchmark portfolio returns were compared before, this paper introduces the period in more than 25 years of long-term bonds and sample concentration of bonds constitute a new combination, to match the duration of the benchmark portfolio, so as to ensure that the pricing error to construct the portfolio and benchmark portfolio based on has the same risk exposure, the two combinations in the investment income is more reasonable.

Because this trading strategy involved in the model contain free parameters, such as moving average model of lag order number  $k$  and the multiplier  $m$ , model and time sequence of multiplier  $m$ . Therefore, this paper will be  $K$  were set for 10 days, 20 days, 30 days and 40 days, two trading signal model by  $m$  were set to 0.5, 1.5, 2 and 2.5 to test based on the pricing errors of the trading strategy of the validity and robustness. In addition, taking into account the 2010 China inter-bank market flow of good bond number 57 and every bond should reflected in the portfolio returns, so in this paper, the capital to limit the proportion of each bond funds portfolio for the total

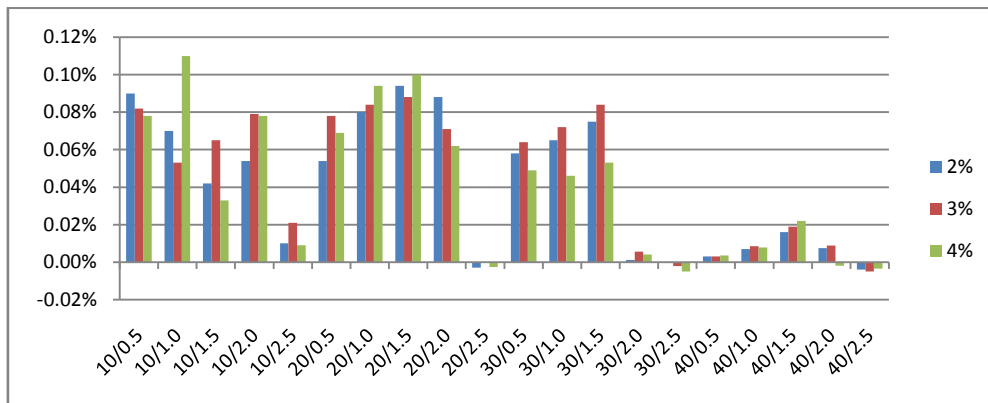
amount of investment is less than or equal to 2%, less than or equal to 3% and less than or equal to 4%, 2% similar to etc. the combination weights, and higher than the 4% the proportion of investment will make money and a few bonds and combination in the rest of the bond investment funds are 0, not only makes the portfolio risk cannot be dispersed effectively, but also makes the most of the bond combination lost the meaning of existence.

#### IV. Results

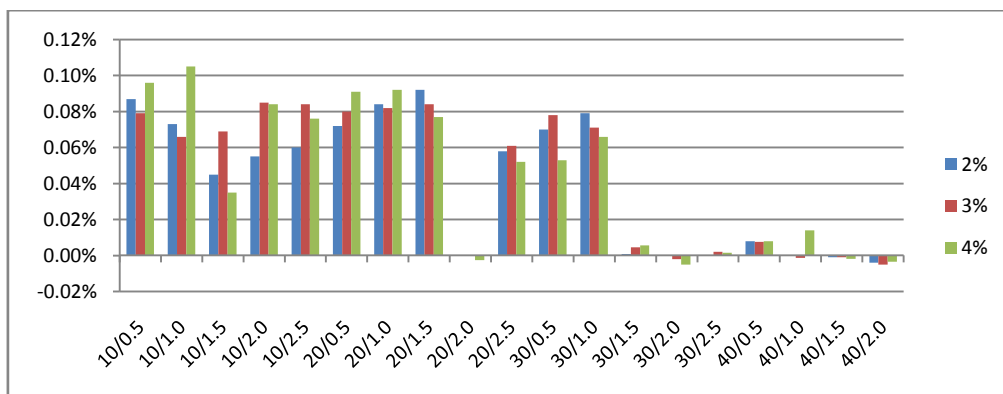
Using the bond market, which is a combination of the bond market of our country, the paper puts forward the bond trading strategy to invest in the portfolio. Figure 1 and Figure 2 show that the design of the trading signals in the moving average model can be compared with the yield of the benchmark portfolio. Obviously, whether for bonds in inter-bank bond index or to buy and hold portfolio maturity, based on the moving average model is constructed for the trading signals, the proportion of investment in a limited to 2%, 3% and 4% and lag period of 10 weeks, 20 weeks and 30 weeks of case can obtain excess return rate of, and for 40 weeks lag, excess return rate is almost zero. This may be due to the lag period is too long, the trading signal insensitive with respect to price changes.

Similarly, in figure 1, the multipliers were 0.5, 1.0, 1.5, 2.0 and 2.5 four cases, time series model by outward prediction pricing errors and build the trading signals, the obtained income rate of more than buy and hold to maturity and bonds in inter-bank bond index reached 11 basis points around.

Further, this paper uses sample concentrated only 57 of the national debt and 3 period to construct a new combination in more than 25 years of national debt, matching buy and hold portfolio and bank debt bond index two benchmark portfolio duration expires, which will be constructed in this paper combination and buy and hold maturities and between bank debt and bond index portfolio risk in the same level. After every bond investment ratio does not exceed 2% of the calculated the trading portfolio and benchmark portfolio in each period the average investment proportion. Table 3 shows the combination is constructed in this paper respectively, and buy and hold portfolio and bank debt bond index portfolio in duration matching under, the investment in 0-3 years, 3-7 years, 7 to 10 years and 10 years four time interval within the funds accounted for the proportion of total investment.



**Figure 1.** Moving average model with respect to the excess return of buying and holding maturity portfolio



**Figure 2.** The excess return of the moving average model with respect to the Treasury bond index

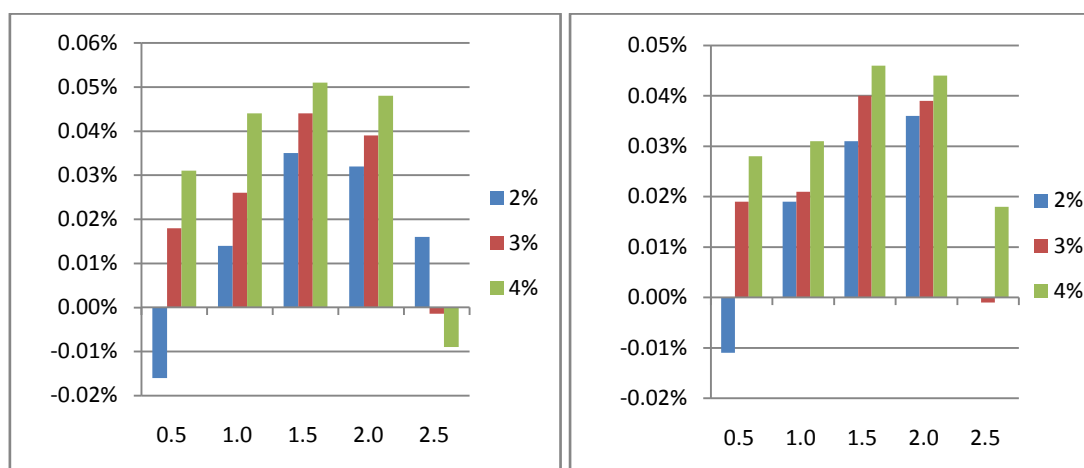


Figure 3. Excess returns relative to the buying and holding maturity portfolio and the Treasury bond index

Table 3. Average investment share of each term in the portfolio

Average investment ratio	0-3 years	3-7 years	7-10 years	>10 years
Portfolio combined with buy-and-hold maturing duration matching				
moving average	9.87%	43.26%	22.67%	24.20%
time series	8.92%	41.37%	22.89%	26.82%
Buy and hold maturity	11.23%	38.55%	18.98%	31.24%
Portfolio combined with debt in the inter-bank bond index duration matching				
moving average	7.78%	40.69%	28.74%	22.79%
time series	6.69%	42.63%	24.51%	26.17%
China Interbank Government Bond Index	6.59%	43.61%	26.67%	23.13%

Obviously, the combination of moving average model and time series model has a certain degree of investment in each period, not only to invest in short-term treasury bonds or long-term treasury bonds. However, due to the composition of the bond index portfolio does not contain 1 year bonds, thus resulting in the debt ratio of the inter-bank bond index portfolio in the 0-3 year period is relatively small. This fully shows that the bond pricing error does contain is not incorporated into the pricing model of effective information, because for any one of the bonds, the effective information of the market are the same, so any one of the bond pricing errors inevitably contain effective market information, therefore short-term bonds, medium-term notes and long-term bond pricing error among are effective market information, and only the market information to any of the role of the market in different period of bond prices, such as the popular and other factors can only cause market in particular maturity of the bond pricing of the existence of bias. So, it is certain that the Chinese government bond pricing error contains effective economic information, and based on this effective information to build a reasonable trading strategy, can get the excess return.

### V. Conclusion And Suggestion

The China inter-bank bond market the bond transaction data information, the term structure of interest rate estimation is obtained by McCulloch cubic spline model and Svensson model and on the basis of China's bond market price and the theoretical price for bond pricing error. After respectively pricing errors to construct bond trading strategies based on and through the China Securities simulated investment income by moving average model and time series model, this paper finds that China inter-bank bond market the bond pricing error contains effective economic information, radicals and on a combination of bond pricing error structure appropriate trading can obtain about 11 basis points of the annual excess return rate.

According to the analysis process and the conclusion, this paper puts forward the following several suggestions: first, as soon as possible to improve the bond market system of China, the end of China's bond market segmentation phenomenon, thereby increasing the various types of bond liquidity; second, the implementation of the bond arbitrage access policy, under the premise of the pricing mechanism is not perfect, through the bond arbitrage mechanism market information will be reflected fully, improve the market is effective.

### References

- [1]. Y.Yifeng, G.Jue, the term structure of interest rates McCulloch three spline estimation method, *statistics and decision*, 17(2012): 67-69.
- [2]. K.Shulong, W.Zhiqiang, The study on the risk characteristics and the information contained in the interest rate term structure of Treasury China, *world economy* , 10(2010):121-142.
- [3]. R.Bliss, Testing term structure estimation methods, *Advances in Futures and Options Research* 9(1997):197-232.
- [4]. E.Elton, and Green, T.C., Tax and Liquidity effects in pricing government bonds, *Journal of Finance* , 53(1998):1533-1562.
- [5]. R.Flavell, N. Meade and G . Salkin, the gilt market: Models and model –based trading. *Journal of the Operational Research Society* 45(1994):392-408.
- [6]. M.Ioannides, A comparison of yield curve estimation techniques using UK data. *Journal of Banking and Finance*, 27(2003):1-26.
- [7]. R. Jankowitsch, Parsimonious estimation of credit spreads. *Journal of Fixed Income* 14(2004):49-63.
- [8]. P. Sercu, and X. Wu, the information content in bond model residuals: An empirical study on the Belgian bond market, *Journal of Banking and Finance* 21(1997): 685-720.
- [9]. J.McCulloch, the tax-adjusted yield curve, *Journal of Finance* 3(1975):811-830.
- [10]. L.Svensson, Estimating and Interpreting the Forward interest rate: Sweden 1992-1994,IMF Working Paper, International Monetary Fund. 1989
- [11]. C.Nelson, and A.F. Siegel, Parsimonious modeling of yield curve, *Journal of Business* 60(1987):473-489.
- [12]. O.Vasicek, and Fong, H.G., term structure modeling using exponential splines, *Journal of Finance* 37(1982):339-348.