Study on Evaluation of Venture Capital Based onInteractive Projection Algorithm

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ABSTRACT：*This paper studies evaluation problem in venture capital. Based on the venture capital and the actual evaluation work, we use an evaluation model proposed by us to evaluate the profitability of enterprises. We establish the impact of investment income and investment risk index system, corresponding to get observational data of the second order indexes. Evaluation model is a kind of generalized linear regression model with convex constraint, in which the dependent variable is unknown and regression coefficients all are calculated in accordance with samples instead of the prior designated. The least squares estimation of the model is given by the interactive projection algorithm between the convex sets, so as to provide a new analysis method for venture capital evaluation index system.*

KEYWORDS: *Venture Capital ;Evaluation Model ; Least Squares Estimation ; Interactive Projection Algorithm*

I. INTRODUCTION

The so-called venture capital assessment is to give an assessment of the benefits and the risk of investment projects according to the commercial prospectus based on the information collection [1]. Evaluation index system is the tool and method of the project evaluation, as a support system for investment appraisal. It links up the investment appraisal and assessment objective organically, and plays a role as a bridge and link, which is indispensable in investment appraisal.Foreign countries have already set up comparatively perfect assessment index system of venture capital. In our country, venture capital is still a new thing, which has not set up comparatively perfect assessment index system yet, so for promoting development of venture capital in China, it is necessary to set up an assessment index system that is suitable for China's actual conditions in venture capital.

II. VENTURE CAPITAL EVALUATION INDEX SYSTEM

The assessment index system of venture capital is an indispensable tool and method in the evaluation process of the venture capital. Scientific and effective assessment index system can help the professional personnel of venture capital to assess venture projects, according to the principle of investment, risk partiality and certain assessment criterion and method. It can exclude projects whose risks are too high or profits are too low among a large number of projects, and choose the investment project that has most appreciable potentiality [2].According to the economic characteristics, we propose the impact factors of China's venture capital assessment and the main index system as shown in Table 1.When we build the index system of venture capital assessment, we can get the data record of each index, and then the index should be gathered. In general, the gathering coefficients or weighted coefficients are assigned beforehand. In this paper we consider the weighted

coefficients are calculated by samples but not man-made. Thus we should introduce a mathematical model for evaluation.

III.   EVALUATION MODEL

In our evaluation model there are m objects to be evaluated, pindexes for evaluation, and n evaluates scores for each object and each index. We need to gather the evaluation score by weighted coefficients $\beta_j \ (j = 1, \cdots p)$ but which are unknown.Pindexes are variables and noted as $x_{(1)}, \cdots x_{(p)}$.

The evaluation value in $(i = 1, \cdots, n) \ i\text{th}$ evaluation for $(k = 1, \cdots, m) \ k\text{th}$ object and by $(j = 1, \cdots p) \ j\text{th}$ index is noted as x_{ijk}. The evaluation value data with p dimension for an object in an evaluation is a row in the data matrix. There are n rows for an object and is a data block. There are m data blocks in the data matrix $X = \{x_{ijk}\}$. Xis a cubic matrix and can be arranged as different form by the regulation of cubic matrix. In this paper we arrange X as $m \times n$ rows with p columns. The score of the last evaluation for each object is $(k = 1, \cdots, m) \ y_k$, and they are unknown also.

The evaluation model with unknown dependent variable is a generalized linear regression model. We must add some constraint for the model to solve it. Of course the weighted coefficients must satisfy $\beta_j \geq 0, \ j = 1, \cdots, p$ (i.e. $\beta \geq 0$), and $+ \beta_p = 1 \ \beta_1 + \beta_2 + \cdots$ (i.e. $\mathbf{1}_p' \beta = 1$). This is a prescription constraint. The dependent variables must also satisfy some constraints. Obviously we only give a unique score for each evaluation object. That is, we only give m score although there $m \times n$ rows in the matrix.   We   define   dependent   variable $Y = Dy$, here $Dy = y \otimes \mathbf{1}_n$, $D_{mn \times m} = I_m \otimes \mathbf{1}_n$, $\mathbf{1}_n = (1, \cdots, 1)'$, $y = (y_1, \cdots, y_m)'$, and $\otimes$ is Kronecker product.

Thus the evaluation model may be expressed as following:

$$Dy = X\beta + \varepsilon \ E(\varepsilon) = 0 \tag{1}$$

$$Var(\varepsilon) = \sigma^2 I \ \mathbf{1}_p' \beta = 1 \ \beta > 0, \ D_{mn \times m} = I_m \otimes \mathbf{1}_n \tag{2}$$

This model is proposed by us in [3]. The data structure of evaluation model is as the Table 2.

IV. THE LEAST SQUARE SOLUTION OF THE GENERALIZED LINEAR REGRESSION MODEL WITH CONVEX CONSTRAINT

We'll discuss the LSE of the evaluation model in three steps [4-6].

Firstly, if Dy is known, it is just an ordinary constraint regression model. Only considering the constraint $\mathbf{1}_p' \beta = 1$, we can get an explicit solution by the method of Lagrange Multiplier. Let

$$Q(\beta, y) = (Dy - X\beta)'(Dy - X\beta) \tag{3}$$

$$\phi(\beta, y) = Q - 2\lambda(\mathbf{1}'_p \beta - 1) \tag{4}$$

where λ be the multiplier. We have

$$\frac{\partial \phi}{\partial \beta} = -2nX'_0 y + 2X'X\beta - 2\lambda\mathbf{1}'_p = 0 \tag{5}$$

$$\frac{\partial \phi}{\partial y} = 2ny - 2nX'_0\beta = 0 \tag{6}$$

where

Investment income index system	
Enterprise Management Level	Enterprise's strategic objectives
	Corporate business culture and philosophy
	The organizational structure of enterprises
	Staff wages and benefits
	Company personnel reserve
Entrepreneurship	Technological innovation ability
	Team control
	Adaptability to the market
	Ability to foresee the risk
	Social interaction
Market Environment	Venture in which industry
	Size of the market
	Market growth
	Market competition
Products and Technical Characteristics	The level of technology patents
	Product adaptation
	Product uniqueness
Earning Capacity	The level of return
	Earnings potential growth
Management Capacity of Enterprises	management decision-making
	The management of enterprise mobility
	Marketing capability
	Adaptability to the market
	Financial risk
Technical Risk	Intellectual property
	Technological advance
	Technology alternatives
	The degree to prevent imitation of technology
	Technology and policy and industry standards compliance
Market Risk	Market stability
	The difficulty of market development
	Competitor status
	Policy trends
Exit Risk	Exit channel status
	The time might withdraw
	The way could pull out

Table 1. Venture capital evaluation index system.

$$X_0' = \frac{1}{n}(I_m \otimes \mathbf{1}_n')X = \frac{1}{n}D'X \tag{7}$$

It is a compression matrix by taking the average value of each column for each data block in matrix $X_{(mn \times p)}$.
Then

$$\hat{y} = X_0'\beta \tag{8}$$

Let

	$x_{(1)}$	$x_{(2)}$	……	$x_{(p)}$
	β_1	β_2	……	β_p
y_1	x_{111}	x_{121}	……	x_{1p1}
y_1	x_{211}	x_{221}	……	x_{2p1}
⋮	⋮	⋮	⋮	⋮
y_1	x_{n11}	x_{n21}	……	x_{np1}
⋮	……	……	……	……
⋮	……	……	……	……
⋮	……	……	……	……
y_m	x_{11m}	x_{12m}	……	x_{1pm}
y_m	x_{21m}	x_{22m}	……	x_{2pm}
⋮	⋮	⋮	⋮	⋮
y_m	x_{n1m}	x_{n2m}	……	x_{npm}

Table 2. The data structure of evaluation model.

$$P_D = I_{mn} - \frac{1}{n}DD' \tag{9}$$

It is easy to verify that P_D is a projection matrix. Since $(X'X - nX_0'X_0)\beta = \lambda\mathbf{1}_p$, letting

$$A = X'X - nX_0'X_0' = X'P_D X \tag{10}$$

when A is invertible, the solution of β is

$$\hat{\beta} = \lambda A^{-1}\mathbf{1}_p' = \frac{A^{-1}\mathbf{1}_p}{\mathbf{1}_p' A^{-1}\mathbf{1}_p'} \tag{11}$$

Summarizing the aforesaid, we have the following theorem.

THEOREM 1. If Dy is known and $rk(P_D X) = p$, under the constrain $\mathbf{1}_p'\beta = 1$,

$$Q(\beta, y) = \| Dy - X\beta \|^2 \xrightarrow{y,\beta} \min(12)$$

has unique solution (8) and (11). If each component of $\hat{\beta}$ is nonnegative, (8) and (11) are also the solution of the evaluation model (1) (2).

Secondly, if some components of $\hat{\beta}$ are negative, we must consider the constraints $\mathbf{1}'_n \beta = 1$ and $\beta > 0$ simultaneously. Then it is a prescription regression model. For the existence and uniqueness of the solution of the model (1) (2) , we have the following theorem.

    THEOREM 2.If $rk(D \mid X) = m + p$, the evaluation model has unique solution.

The proof is easy when we rewrite (12) as

$$\left\| 0 - (D \vdots X)\begin{pmatrix} -y \\ \beta \end{pmatrix} \right\|^2 \xrightarrow{y,\beta} \min \tag{13}$$

the set

$$A_{DX} = \{ (D \vdots X)\begin{pmatrix} -y \\ \beta \end{pmatrix} \mid y \in R^m,$$
$$\mathbf{1}'_p \beta = 1, \beta \geq 0, \beta \in R^p \} \tag{14}$$

is a closed set, so there exists an unique point $A_0 \in A_{DX}$ and A_0 satisfies (13). Because the column of $(D \mid X)$ is full rank, we can obtain unique solution of y and β from $A_0 = X\beta - Dy$.

Thirdly, we'll discuss the algorithm of the model when Dy is unknown. We consider the geometric background of (12). Denote sets

$$A = \{ Dy \mid y \in R^m \} \tag{15}$$
$$B = \{ X\beta \mid \mathbf{1}'_p \beta = 1, \beta \geq 0, \beta \in R^p \} \tag{16}$$

The Formula (12) means to seek the shortest Euclidean distance between sets A and B .

Obviously, A and B are two closed convex sets and B is bounded. According to the theorem "the distance between two closed convex sets can be reached", the solution of (12) exists.

How to find the shortest distance between two convex sets? We consider the method of the alternating projection between two sets. Let A_0 be a point in the set A . If $d(A_0, B_0) = d(A_0, B)$, we call B_0 the projection from the point A_0 to the set B . The following is the alternating projection process.

Take an arbitrary initial value $A_0 \in A$, find $B_0 \in B$, satisfying $d(A_0, B_0) = d(A_0, B)$. For B_0, take $A_1 \in A$, satisfying $d(B_0, A_1) = d(B_0, A)$. For $A_i \in A$, take $B_1 \in B$, satisfying $d(A_i, B_i) = d(A_i, B)$. For B_i, take $A_{i+1} \in A$, satisfying $d(B_i, A_{i+1}) = d(B_i, A)$, and so on. When $d(A_i, B_i) < \varepsilon$, iterative process is stopped and computation is completed. The meaning of convergence of aforesaid iterative process is:

$$\lim_{i \to \infty} d(A_i, B_i) = d(A, B) = d(A^*, B^*)$$
(17)

where A^*, B^* are two points which belong to sets A and B respectively.

In one word, the distance between two closed convex sets may be obtained by making use of successive computation of distance between a point and a closed convex set. In the evaluation model consisting of (1) (2), for arbitrary y_i, we can get the solution of β_i according to the Theorem 1 and 2. For the solution of β_i, (1) becomes a common multivariate regression model which can be solved. The convergence of the alternating projection iterative process has been proved in [3] and is omitted here.

5. Example

The following example is given by the DASC software developed by ourselves. The evaluation model data has 30 rows and 6 columns. Evaluation marks are all set from 1 to 10. The matrix of evolution data is as the **Table 3**.

The computation process shows the convergence process is very fast and it has iterated 7 times altogether. The subprogram gives an initial value $y_1 = y_2 = y_3 = 10$ and a control precision 0.0001. The sum of regression coefficients should be 1 in the printing result of iterative process.

Notice that the three different fitted values of y are the evaluation results actually. And the observation value of y is calculated according to fitting equation. The fitting effect figure provides a visual awareness of

	β_1	β_2	β_3	β_4	β_5	β_6
Y_1	1	2	8	5	6	3
Y_1	1	7	7	10	4	6
Y_1	9	1	1	6	7	1
Y_1	4	1	5	7	6	10
Y_1	9	6	1	7	5	8
Y_1	10	8	3	1	8	4
Y_1	7	8	10	4	3	10
Y_1	8	8	7	1	7	9
Y_1	3	5	8	5	3	3
Y_1	4	2	5	9	10	1
Y_2	10	6	6	4	10	5
Y_2	3	1	10	1	6	4
Y_2	3	10	6	5	10	1
Y_2	8	8	9	2	1	7

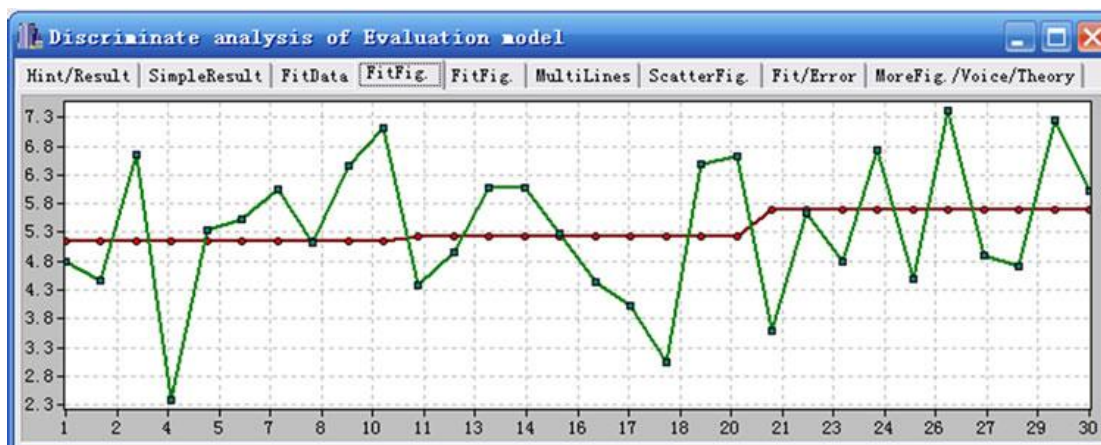|						
Y_2	9	7	8	8	10	9
Y_2	3	4	4	6	6	9
Y_2	5	9	3	5	6	5
Y_2	3	2	2	6	9	1
Y_2	6	5	10	8	6	9
Y_2	7	9	2	3	8	2
Y_3	1	3	1	5	1	8
Y_3	10	3	2	4	9	7
Y_3	2	7	4	4	5	2
Y_3	6	9	6	10	6	2
Y_3	10	5	2	6	3	5
Y_3	5	10	2	2	4	7
Y_3	2	7	7	9	3	5
Y_3	4	3	1	10	5	2
Y_3	10	5	2	9	1	2
Y_3	1	4	3	2	8	5

Table 3.The martix of evalution data.

Figure 1.The fitting effect figure.

evaluation model. Results are y = 5.1451, 5.2407, 5.6958), β = (0.2298, 0.2106, 0.1510, 0.0588, 0.2457, 0.1041), where y is the final evaluation mark and β is the regression coefficient, also called weight coefficient.

Evaluation model can be used as the basis for clustering. When we input X = (2, 3, 4, 5, 6, 7) in the prediction program, the forecast value is 5.1190. It denotes that this group data should attribute to the fist group, because the difference between 5.1190 and 5.1451 is the minimum.

V. CONCLUSIONS

In this paper, we studied the index system of risk evaluation for venture capital, and gave the algorithms of model in which the gathering coefficients of index were calculated by samples. Our algorithm also provides a solution scheme for some problems in venture capital evaluation, so this paper gives a new idea for venture capital evaluation with some practical reference value.

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